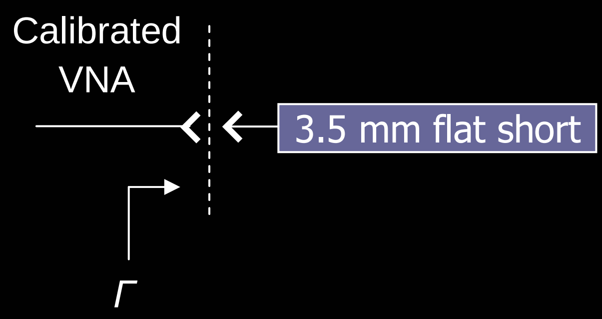
Covariance-Based Uncertainty Representation for Frequency & Time Domain Measurements

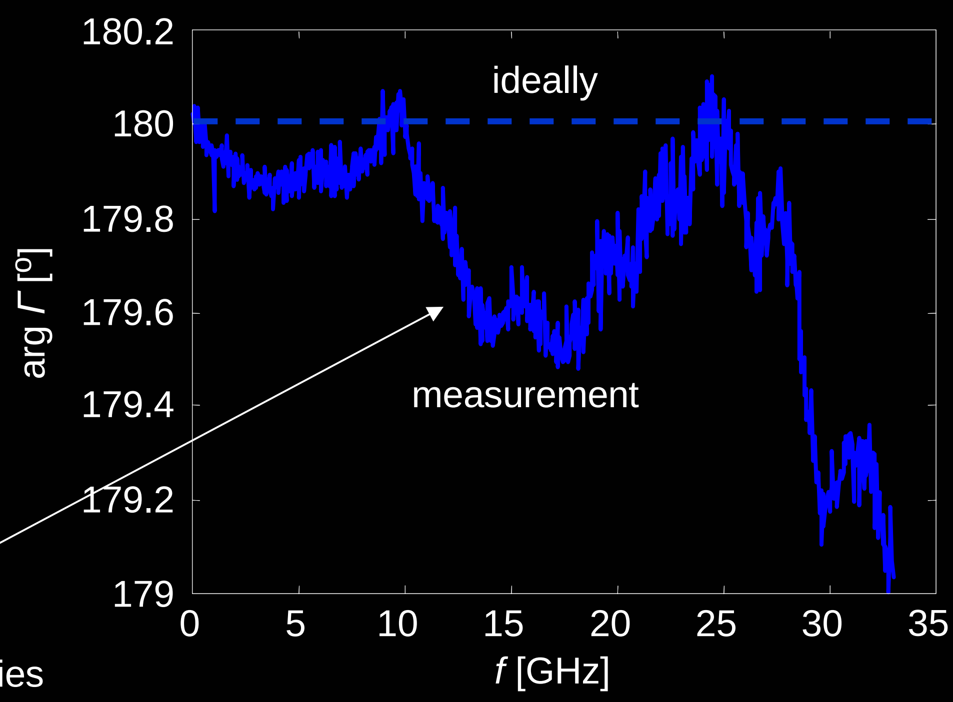
Arkadiusz Lewandowski

National Institute of Standards and Technology
Guest Researcher

NVNA Users' Forum, San Francisco, June 2006

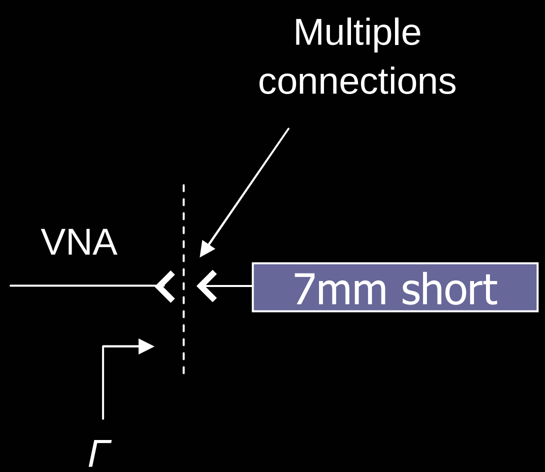
Example of uncertainty correlation in frequency

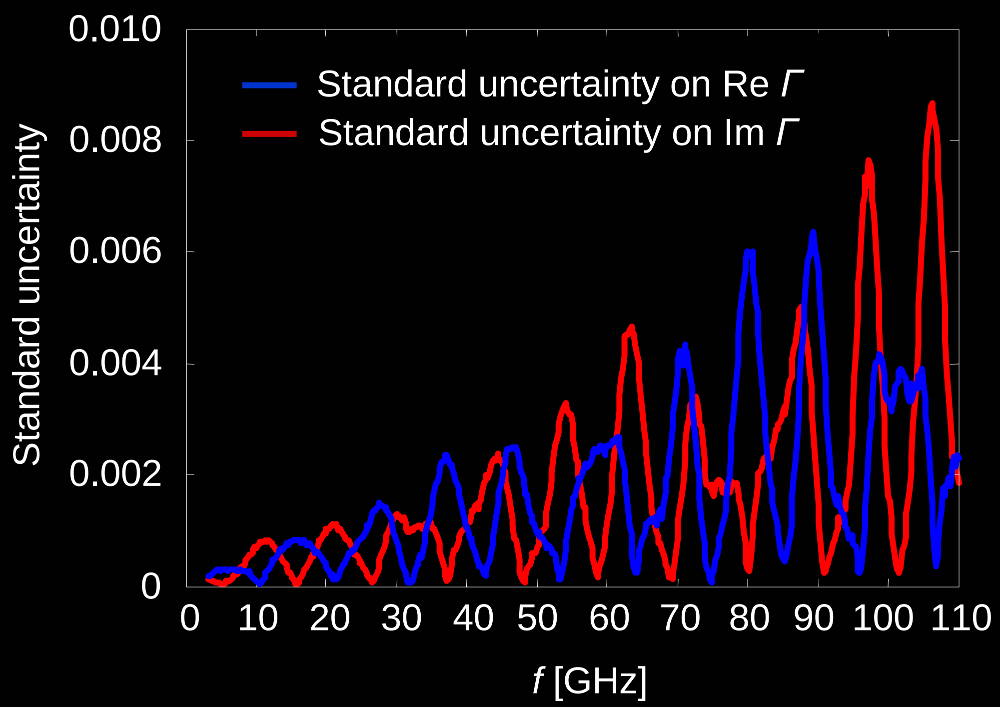




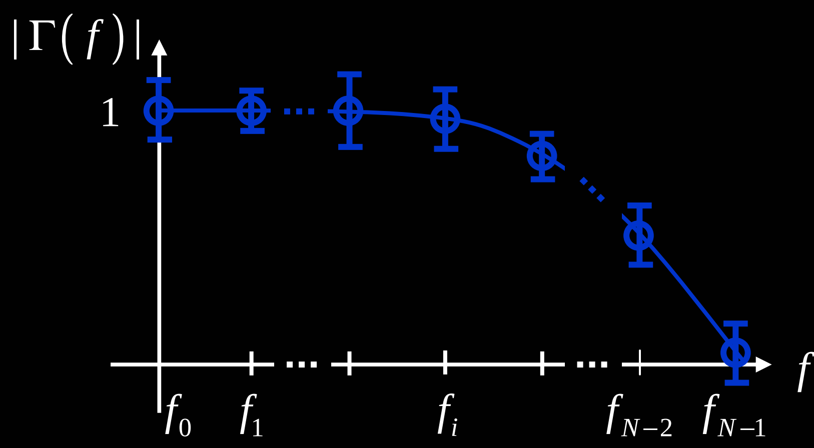
Calibration uncertainties
for different frequencies
are correlated!

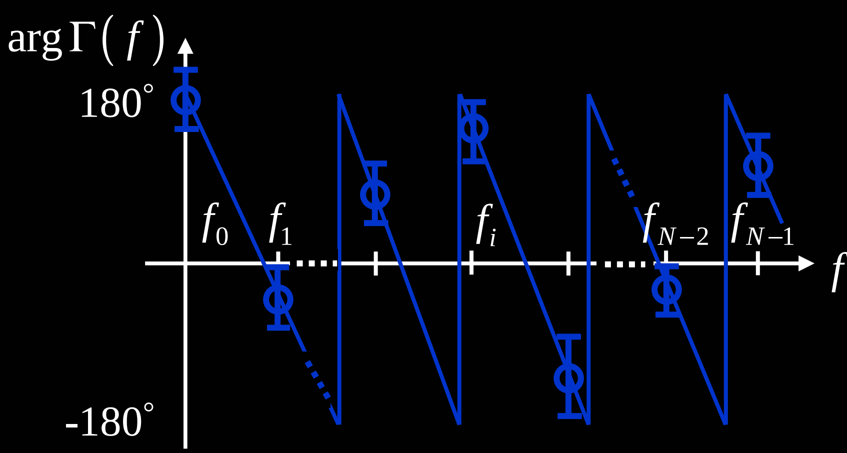
Example of correlations in measurement uncertainties





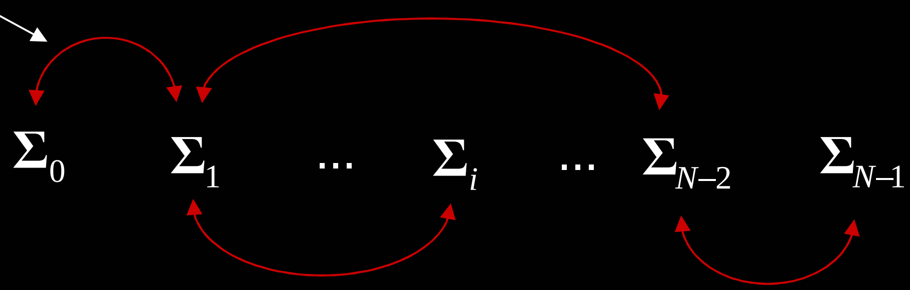
Typical error representation for S-parameter measurements



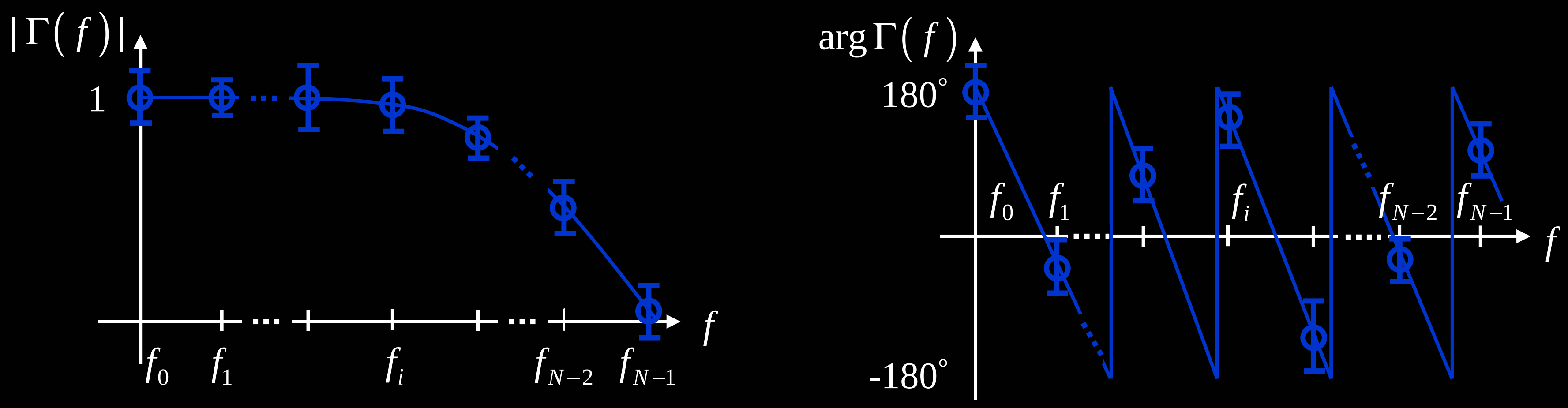


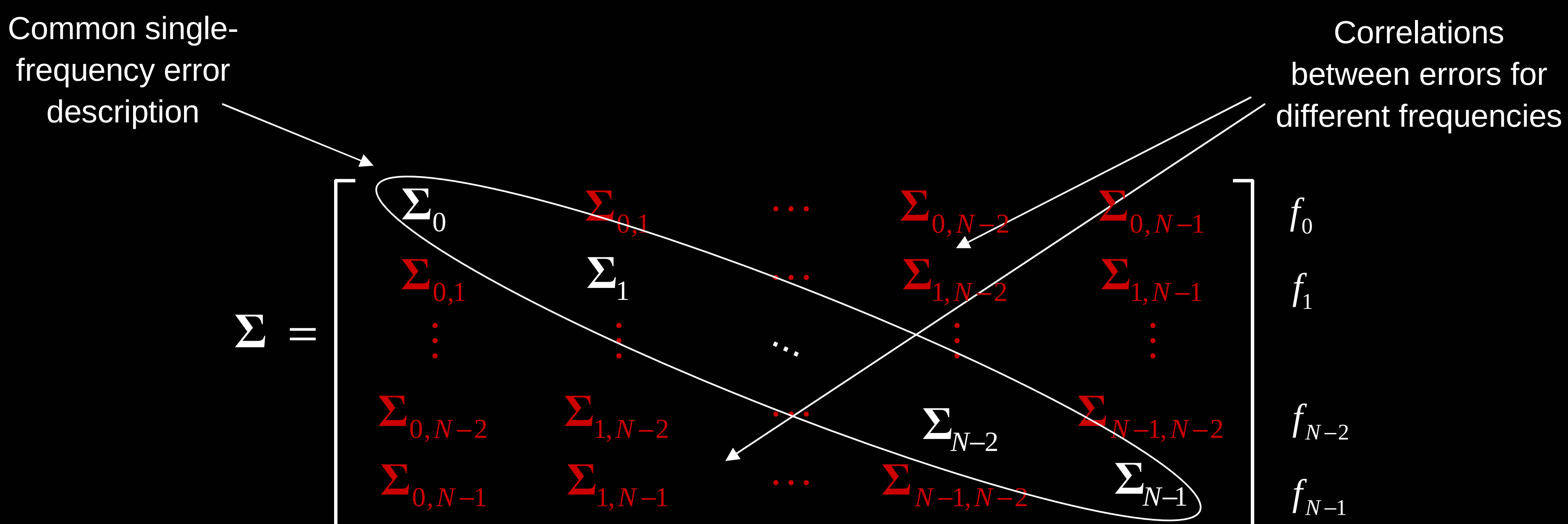
$$\Sigma_i = \begin{bmatrix} \sigma^2_{\mathrm{Re}\Delta\Gamma} & \rho\sigma_{\mathrm{Re}\Delta\Gamma}\sigma_{\mathrm{Im}\Delta\Gamma} \\ \rho\sigma_{\mathrm{Re}\Delta\Gamma}\sigma_{\mathrm{Im}\Delta\Gamma} & \sigma^2_{\mathrm{Im}\Delta\Gamma} \end{bmatrix}$$

Correlations between uncertainties for different frequencies NOT included



Including error correlations between different frequencies



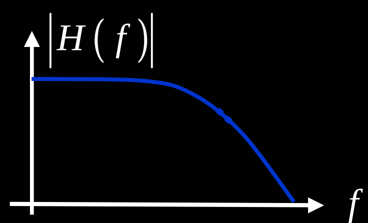


$$\Sigma = \begin{bmatrix} \Sigma_0 & \Sigma_{0,1} & \cdots & \Sigma_{0,N-2} & \Sigma_{0,N-1} \\ \Sigma_{0,1} & \Sigma_1 & \cdots & \Sigma_{1,N-2} & \Sigma_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Sigma_{0,N-2} & \Sigma_{1,N-2} & \cdots & \Sigma_{N-2} & \Sigma_{N-1,N-2} \\ \Sigma_{0,N-1} & \Sigma_{1,N-1} & \cdots & \Sigma_{N-1,N-2} & \Sigma_{N-1} \end{bmatrix} \begin{matrix} f_0 \\ f_1 \\ \\ f_{N-2} \\ f_{N-1} \end{matrix}$$

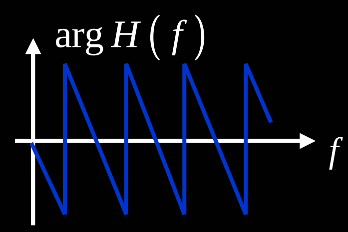
$$\begin{matrix} f_0 & f_1 & & f_{N-2} & f_{N-1} \end{matrix}$$

Consequences for time-domain characterization

Measured complex spectrum



&

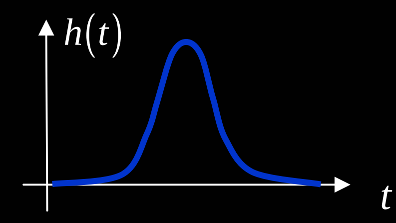


$$\mathbf{H} = \begin{bmatrix} \mathrm{Re}\, H(0) \\ \mathrm{Im}\, H(0) \\ \vdots \\ \mathrm{Re}\, H(f_{N-1}) \\ \mathrm{Im}\, H(f_{N-1}) \end{bmatrix} + \widetilde{\Delta\mathbf{H}}$$

Inverse discrete Fourier transform (IDFT)

$$\mathbf{h} = \mathbf{W}\mathbf{H}$$

Waveform



$$\mathbf{h} = \begin{bmatrix} h(t_0) \\ \vdots \\ h(t_{N-1}) \end{bmatrix} + \widetilde{\Delta\mathbf{h}}$$

Uncertainty propagation through IDFT

$$\widetilde{\Delta\mathbf{h}} = \mathbf{W}\,\widetilde{\Delta\mathbf{H}}$$

Consequences for time-domain measurements

Uncertainty propagation
through IDFT

$$\Delta\tilde{\mathbf{h}} = \mathbf{W}\;\Delta\tilde{\mathbf{H}}$$

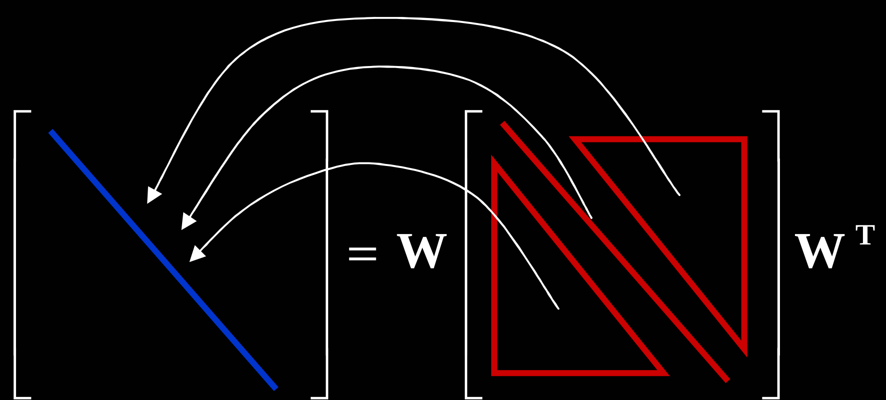
Covariance matrix for time-domain result

$$\boldsymbol{\Sigma}_{\mathbf{h}} = E\left\{\Delta\tilde{\mathbf{h}}\;\Delta\tilde{\mathbf{h}}^{T}\right\}$$

Covariance matrix for spectrum measurement errors

$$\boldsymbol{\Sigma}_{\mathbf{H}} = E\left\{\Delta\tilde{\mathbf{H}}\;\Delta\tilde{\mathbf{H}}^{T}\right\}$$

$$\boldsymbol{\Sigma}_{\mathbf{h}} = \mathbf{W}\;\boldsymbol{\Sigma}_{\mathbf{H}}\,\mathbf{W}^{T}$$

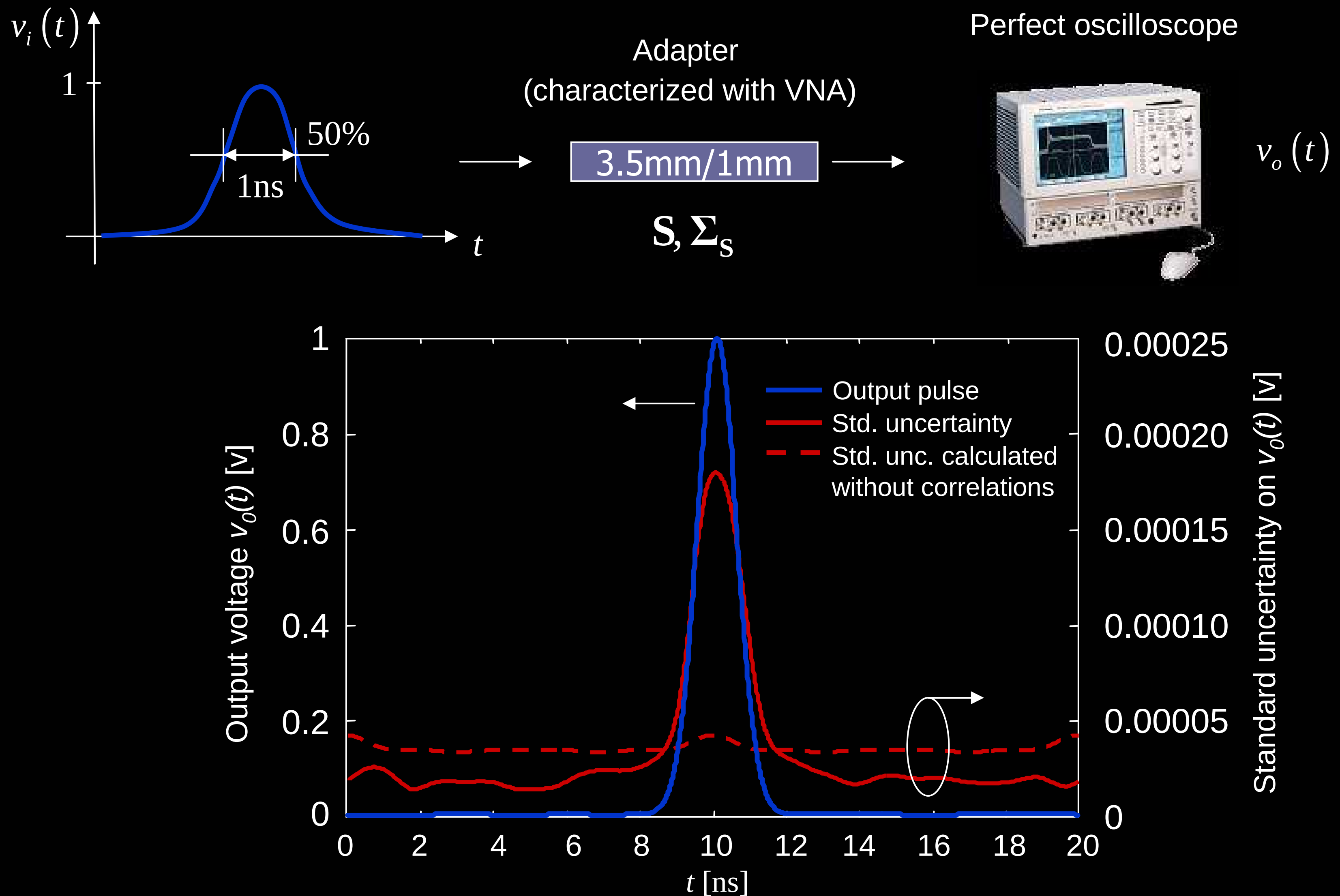


Correlations between uncertainties in FD contribute to standard uncertainties in TD !!!

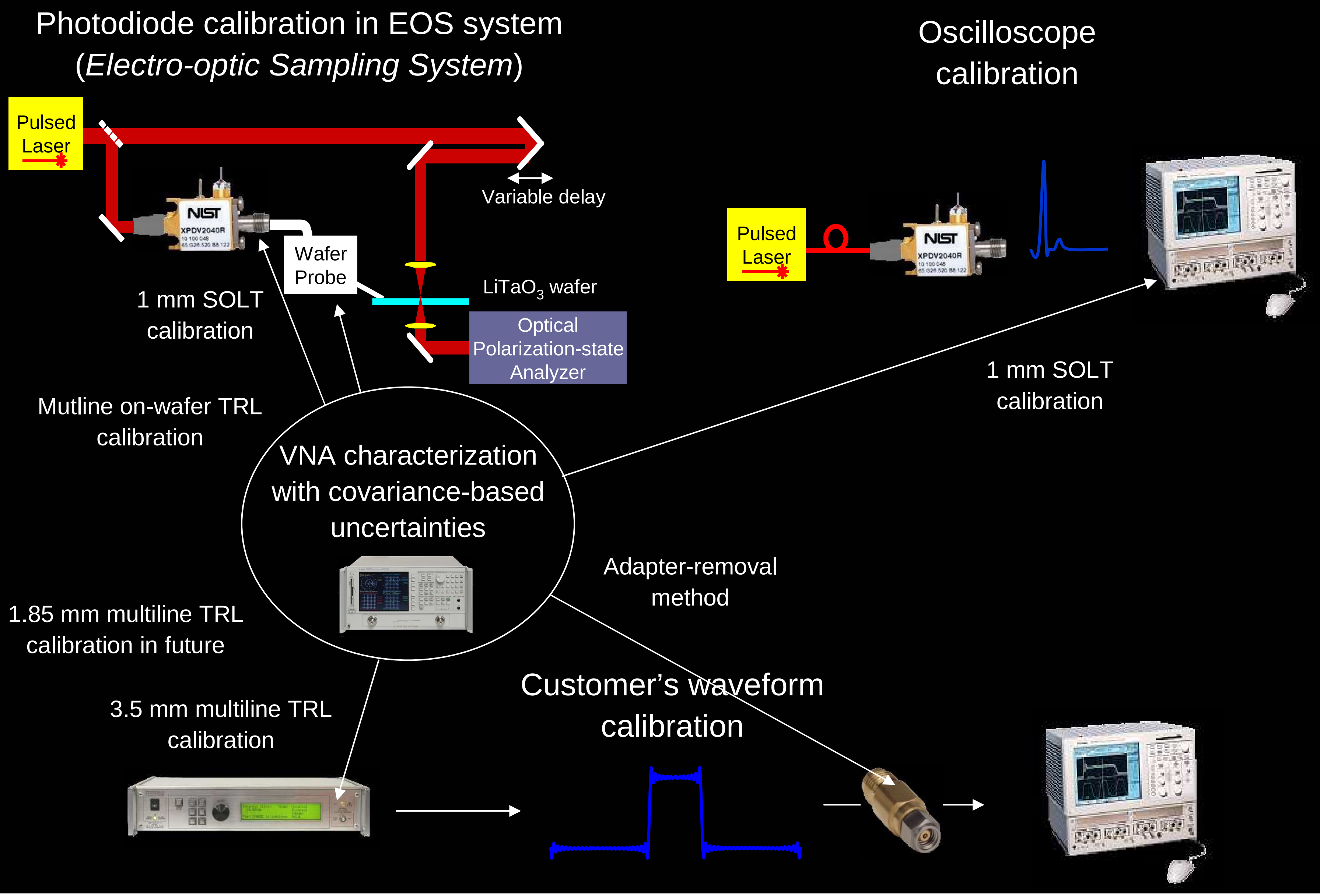
⇩

Need to be accounted for !!!

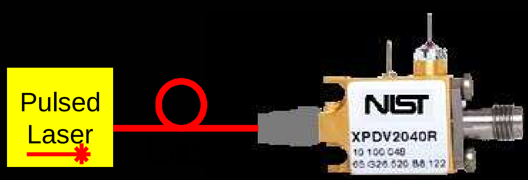
Example: Adapter effect in time-domain

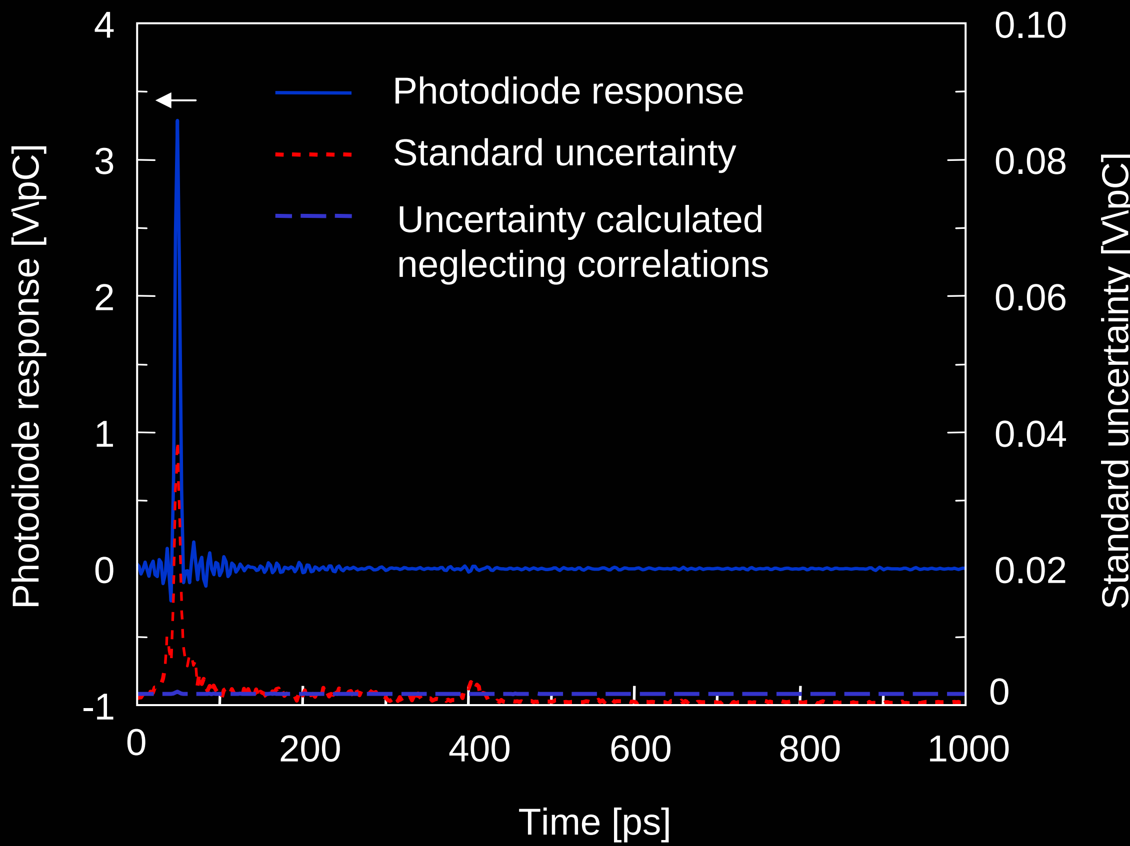


Example: NIST 110 GHz waveform calibration service



Example: NIST 110 GHz waveform calibration service





Consequences for measurement-based modeling

Measurement
(e.g. *S*-parameters)

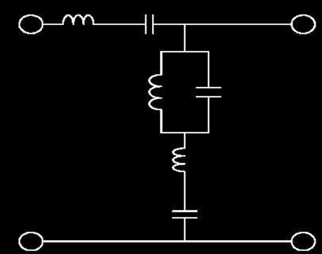
$$\mathbf{m} = \begin{bmatrix} \mathbf{m}_{f_0} \\ \mathbf{m}_{f_1} \\ \vdots \\ \mathbf{m}_{f_{N-1}} \end{bmatrix} + \Delta\tilde{\mathbf{m}}$$

modeling

$$\mathbf{p} = \mathbf{f}(\mathbf{m})$$

$$\mathbf{p} = \begin{bmatrix} L_1 \\ C_1 \\ \vdots \end{bmatrix} + \Delta\tilde{\mathbf{p}}$$

Equivalent circuit



Uncertainty propagation
through modeling

$$\Delta\tilde{\mathbf{p}} \approx \frac{\partial \mathbf{f}(\mathbf{m})}{\partial \mathbf{m}^T} \Delta\tilde{\mathbf{m}} = \mathbf{J}\,\Delta\tilde{\mathbf{m}}$$

Matrix of sensitivity
coefficients (Jacobian)

Consequences for measurement-based modeling

Uncertainty propagation
through modeling procedure

$$\Delta\tilde{\mathbf{p}} \approx \frac{\partial \mathbf{f}(\mathbf{m})}{\partial \mathbf{m}^T} \Delta\tilde{\mathbf{m}} = \mathbf{J}\,\Delta\tilde{\mathbf{m}}$$

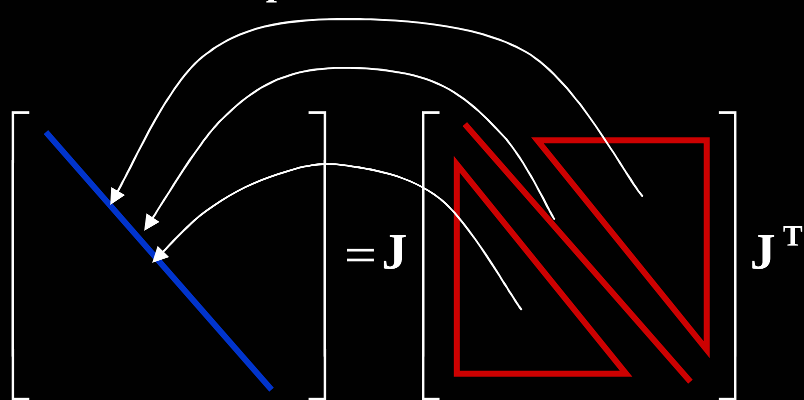
Covariance matrix for model parameter errors

$$\mathbf{\Sigma}_{\mathbf{p}} = E\left\{\Delta\tilde{\mathbf{p}}\,\Delta\tilde{\mathbf{p}}^T\right\}$$

Covariance matrix for S-parameter measurement errors

$$\mathbf{\Sigma}_{\mathbf{m}} = E\left\{\Delta\tilde{\mathbf{m}}\,\Delta\tilde{\mathbf{m}}^T\right\}$$

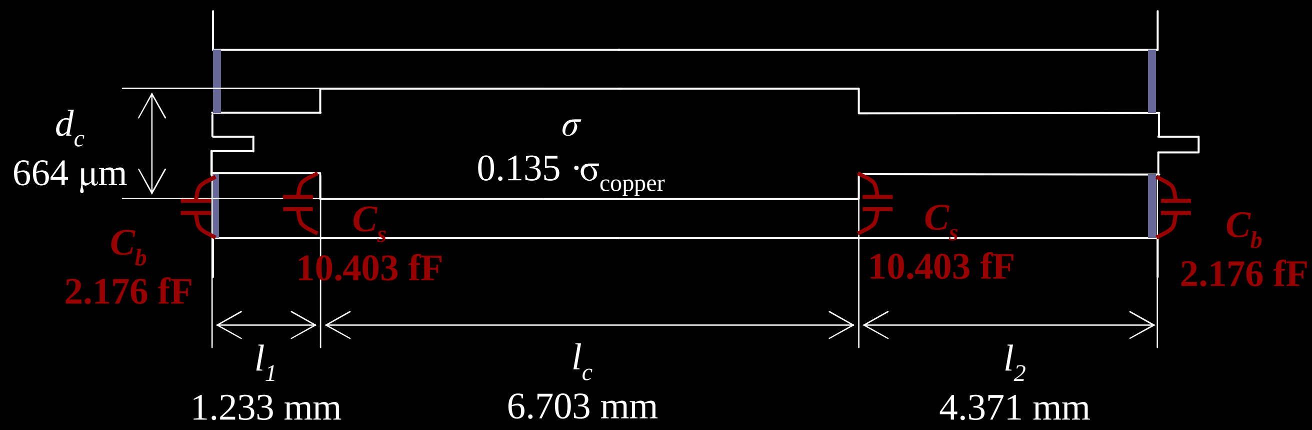
$$\mathbf{\Sigma}_{\mathbf{p}} \approx \mathbf{J}\,\mathbf{\Sigma}_{\mathbf{m}}\mathbf{J}^T$$

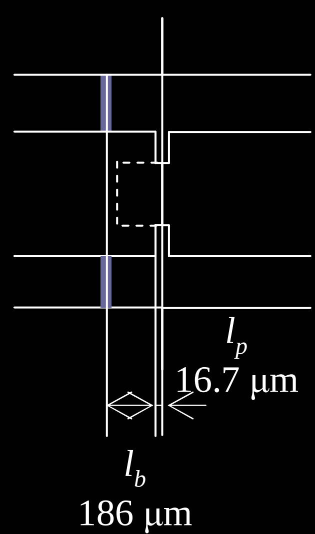


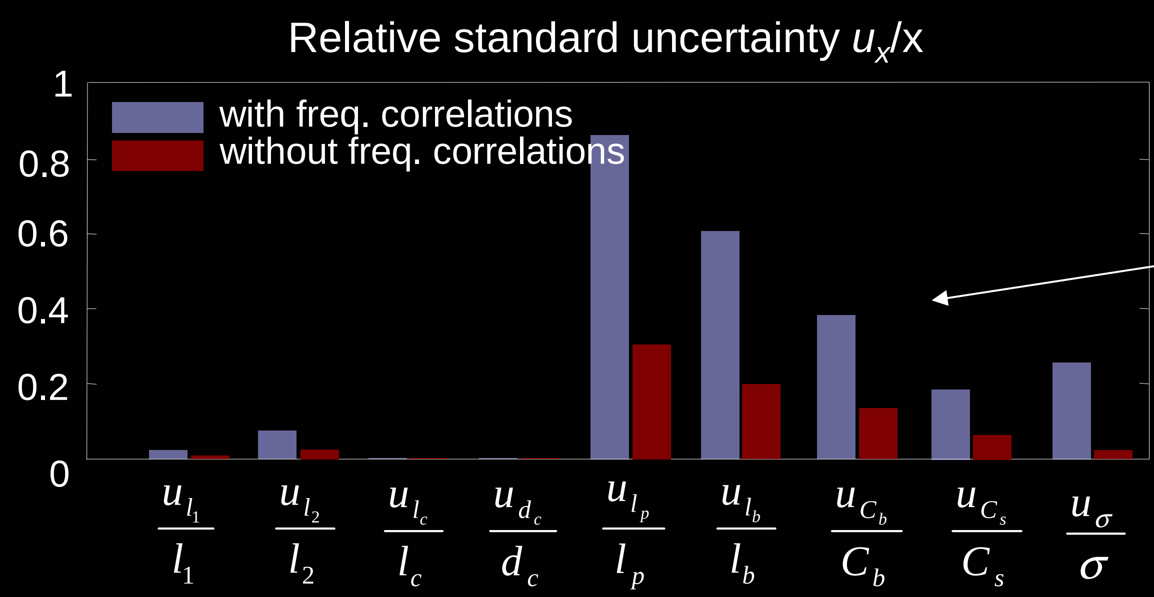
Correlations between uncertainties in FD contribute to standard uncertainties of model parameters !!!

Need to be accounted for !!!

Example: Modeling 1 mm mismatch-thru standard







Neglecting frequency correlations leads to underestimation of model parameter uncertainties !!!

Summary

- Correlation between measurements uncertainties for different frequencies and times possible
- Generalized covariance-matrix approach as a means for systematic description of these correlations
- Correct description of these correlations crucial for
  - Time-domain characterization involving VNA measurements
  - Measurement-based modeling
- Covariance-matrix approach sucesfully applied to capture these correlations in VNA measurements used in the 110 GHZ NIST waveform calibration service
- Other possible applications
  - Time-domain reflectometry
  - Link simulation based on S-parameters (eye-diagrams, BER)
  - Uncertainty analysis for LSNA measurements